Custodial symmetry breaking in the two-Higgs-doublet model (2HDM)

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This talk is based on work that appears in:

1. S. Davidson and H.E. Haber, "Basis-independent methods for the two-Higgs-doublet model," *Phys. Rev.* **D72**, 035004 (2005).

2. H.E. Haber and D. O'Neil, "Basis-independent methods for the two-Higgs-doublet model. II: The significance of $\tan\beta$," *Phys. Rev.* **D74**, 015018 (2006).

3. H.E. Haber and D. O'Neil, "Basis-independent methods for the two-Higgs-doublet model. III: CP-violation, custodial symmetry, and the oblique parameters S, T and U," (in preparation).

Outline

- The basis-independent formulation of the 2HDM
- The Higgs basis and Higgs mass-eigenstate basis
- Conditions for neutral-Higgs CP-violation
- Identifying the CP-odd Higgs scalar
- Custodial symmetry in the 2HDM
- 2HDM contributions to S, T and U
- Lessons for future work

The General Two-Higgs-Doublet Model

Consider the 2HDM potential in a *generic* basis:

$$\begin{aligned}\mathcal{V} &= m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - [m_{12}^2\Phi_1^\dagger\Phi_2 + \text{h.c.}] + \tfrac{1}{2}\lambda_1(\Phi_1^\dagger\Phi_1)^2 \\ &\quad + \tfrac{1}{2}\lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\ &\quad + \left\{\tfrac{1}{2}\lambda_5(\Phi_1^\dagger\Phi_2)^2 + \left[\lambda_6(\Phi_1^\dagger\Phi_1) + \lambda_7(\Phi_2^\dagger\Phi_2)\right]\Phi_1^\dagger\Phi_2 + \text{h.c.}\right\}\end{aligned}$$

A basis change consists of a U(2) transformation $\Phi_a \to U_{a\bar{b}}\Phi_b$ (and $\Phi_{\bar{a}}^\dagger = \Phi_{\bar{b}}^\dagger U_{b\bar{a}}^\dagger$). Rewrite $\mathcal{V}$ in a U(2)-covariant notation:

$$\boxed{\mathcal{V} = Y_{a\bar{b}}\Phi_{\bar{a}}^\dagger\Phi_b + \tfrac{1}{2}Z_{a\bar{b}c\bar{d}}(\Phi_{\bar{a}}^\dagger\Phi_b)(\Phi_{\bar{c}}^\dagger\Phi_d)}$$

where $Z_{a\bar{b}c\bar{d}} = Z_{c\bar{d}a\bar{b}}$ and hermiticity implies $Y_{a\bar{b}} = (Y_{b\bar{a}})^*$ and $Z_{a\bar{b}c\bar{d}} = (Z_{b\bar{a}d\bar{c}})^*$. The barred indices help keep track of which indices transform with U and which transform with $U^\dagger$. For example, $Y_{a\bar{b}} \to U_{a\bar{c}}Y_{c\bar{d}}U_{d\bar{b}}^\dagger$ and $Z_{a\bar{b}c\bar{d}} \to U_{a\bar{e}}U_{f\bar{b}}^\dagger U_{c\bar{g}}U_{h\bar{d}}^\dagger Z_{e\bar{f}g\bar{h}}$.

The most general U(1)$_{\rm EM}$-conserving vacuum expectation value (vev) is:

$$\langle\Phi_a\rangle=\frac{v}{\sqrt{2}}\left(\begin{array}{c}0\\ \widehat{v}_a\end{array}\right)\,,\qquad\text{with}\qquad \widehat{v}_a\equiv e^{i\eta}\left(\begin{array}{c}c_\beta\\ s_\beta\,e^{i\xi}\end{array}\right)\,,$$

where $v\equiv 2m_W/g=246$ GeV. The overall phase η is arbitrary (and can be removed with a U(1)$_{\rm Y}$ hypercharge transformation). If we define the hermitian matrix $V_{a\bar{b}}\equiv\hat{v}_a\hat{v}^*_{\bar{b}}$, then the scalar potential minimum condition is given by the invariant condition:

$${\rm Tr}\,(VY)+\tfrac{1}{2}v^2 Z_{a\bar{b}c\bar{d}}V_{b\bar{a}}V_{d\bar{c}}=0\,.$$

The orthonormal eigenvectors of $V_{a\bar{b}}$ are $\hat{v}_b$ and $\widehat{w}_b\equiv\widehat{v}^*_{\bar{c}}\,\epsilon_{cb}$ (with $\epsilon_{12}=-\epsilon_{21}=1$, $\epsilon_{11}=\epsilon_{22}=0$). Note that $\hat{v}^*_{\bar{b}}\hat{w}_b=0$. Under a U(2) transformation, $\hat{v}_a\to U_{a\bar{b}}\hat{v}_b$, but:

$$\widehat{w}_a\to(\det\,U)^{-1}\,U_{a\bar{b}}\,\widehat{w}_b\,,$$

where $\det\,U\equiv e^{i\chi}$ is a pure phase. That is, $\widehat{w}_a$ is a pseudo-vector with respect to U(2). One can use $\widehat{w}_a$ to construct a proper second-rank tensor: $W_{a\bar{b}}\equiv\hat{w}_a\hat{w}^*_{\bar{b}}\equiv\delta_{a\bar{b}}-V_{a\bar{b}}$.

Remark: U(2) $\cong$ SU(2)$\times$U(1)$_{\rm Y}/\mathbb{Z}_2$. The parameters m_{11}^2, m_{22}^2, m_{12}^2, and $\lambda_1,\ldots,\lambda_7$ are invariant under U(1)$_{\rm Y}$ transformations, but change under a "flavor"-SU(2) transformation; whereas $\hat{v}$ transforms under the full U(2) group.

A list of invariant and pseudo-invariant quantities

$$Y_1 \equiv \mathrm{Tr}\,(YV)\,, \qquad Y_2 \equiv \mathrm{Tr}\,(YW)\,,$$

$$Z_1 \equiv Z_{a\bar{b}c\bar{d}}\,V_{b\bar{a}}V_{d\bar{c}}\,, \qquad Z_2 \equiv Z_{a\bar{b}c\bar{d}}\,W_{b\bar{a}}W_{d\bar{c}}\,,$$

$$Z_3 \equiv Z_{a\bar{b}c\bar{d}}\,V_{b\bar{a}}W_{d\bar{c}}\,, \qquad Z_4 \equiv Z_{a\bar{b}c\bar{d}}\,V_{b\bar{c}}W_{d\bar{a}}$$

are invariants, whereas the following (potentially complex) pseudo-invariants

$$Y_3 \equiv Y_{a\bar{b}}\,\widehat{v}_{\bar{a}}^{\,*}\,\widehat{w}_b\,, \qquad Z_5 \equiv Z_{a\bar{b}c\bar{d}}\,\widehat{v}_{\bar{a}}^{\,*}\,\widehat{w}_b\,\widehat{v}_{\bar{c}}^{\,*}\,\widehat{w}_d\,,$$

$$Z_6 \equiv Z_{a\bar{b}c\bar{d}}\,\widehat{v}_{\bar{a}}^{\,*}\,\widehat{v}_b\,\widehat{v}_{\bar{c}}^{\,*}\,\widehat{w}_d\,, \qquad Z_7 \equiv Z_{a\bar{b}c\bar{d}}\,\widehat{v}_{\bar{a}}^{\,*}\,\widehat{w}_b\,\widehat{w}_{\bar{c}}^{\,*}\,\widehat{w}_d\,.$$

transform as

$$[Y_3, Z_6, Z_7] \to (\det\, U)^{-1}[Y_3, Z_6, Z_7] \quad \text{and} \quad Z_5 \to (\det\, U)^{-2} Z_5\,.$$

Physical quantities must be invariants. For example, the charged Higgs boson mass is $m_{H^\pm}^2 = Y_2 + \frac{1}{2}Z_3 v^2$. Pseudo-invariants are useful because one can always combine two such quantities to create an invariant.

The significance of the Higgs basis

Define new Higgs-doublet fields: $H_1 \equiv \hat{v}^*_{\hat{a}} \Phi_a$ and $H_2 \equiv \hat{w}^*_{\hat{a}} \Phi_a$. Then, the Higgs basis is defined such that

$$\langle H_1^0 \rangle = v/\sqrt{2}\,, \qquad \langle H_2^0 \rangle = 0\,,$$

where $v = 246$ GeV. Note that H_1^0 is an invariant field, where H_2^0 is pseudo-invariant (corresponding to a possible rephasing of H_2). The Higgs potential in this basis is:

$$\begin{aligned}
\mathcal{V} &= Y_1 H_1^\dagger H_1 + Y_2 H_2^\dagger H_2 + [Y_3 H_1^\dagger H_2 + \text{h.c.}] \\
&\quad + \tfrac{1}{2} Z_1 (H_1^\dagger H_1)^2 + \tfrac{1}{2} Z_2 (H_2^\dagger H_2)^2 + Z_3 (H_1^\dagger H_1)(H_2^\dagger H_2) + Z_4 (H_1^\dagger H_2)(H_2^\dagger H_1) \\
&\quad + \left\{ \tfrac{1}{2} Z_5 (H_1^\dagger H_2)^2 + \left[Z_6 (H_1^\dagger H_1) + Z_7 (H_2^\dagger H_2) \right] H_1^\dagger H_2 + \text{h.c.} \right\}\,,
\end{aligned}$$

where the coefficients of $\mathcal{V}$ correspond to the (pseudo-)invariants introduced previously. The potential minimum conditions are: $Y_1 = -\frac{1}{2} Z_1 v^2$ and $Y_3 = -\frac{1}{2} Z_6 v^2$.

Example: for the MSSM Higgs sector,

$$Z_1 = Z_2 = \tfrac{1}{4}(g^2 + g'^2)\cos^2 2\beta\,, \qquad Z_3 = Z_5 + \tfrac{1}{4}(g^2 - g'^2)\,, \qquad Z_4 = Z_5 - \tfrac{1}{2} g^2\,,$$

$$Z_5 = \tfrac{1}{4}(g^2 + g'^2)\sin^2 2\beta\,, \qquad Z_7 = -Z_6 = \tfrac{1}{4}(g^2 + g'^2)\sin 2\beta \cos 2\beta\,.$$

The Higgs mass-eigenstate basis

The three physical neutral Higgs boson mass-eigenstates are determined by diagonalizing a 3×3 real symmetric squared-mass matrix that is defined in a basis in which only one of the neutral Higgs bosons has a vacuum expectation value (the so-called "Higgs basis"). The diagonalizing matrix is a 3×3 real orthogonal matrix that depends on three angles: θ_{12}, θ_{13} and θ_{23}. Under a U(2) transformation,

$$\theta_{12}\,,\ \theta_{13}\ \text{are invariant, and} \quad e^{i\theta_{23}} \to (\det U)^{-1} e^{i\theta_{23}}\,.$$

One can express the mass eigenstate neutral Higgs directly in terms of the original shifted neutral fields, $\overline{\Phi}^0_a \equiv \Phi^0_a - v\widehat{v}_a/\sqrt{2}$:

$$h_k = \frac{1}{\sqrt{2}}\left[\overline{\Phi}^{0\,\dagger}_{\bar{a}}(q_{k1}\widehat{v}_a + q_{k2}\widehat{w}_a e^{-i\theta_{23}}) + (q^*_{k1}\widehat{v}^*_{\bar{a}} + q^*_{k2}\widehat{w}^*_{\bar{a}} e^{i\theta_{23}})\overline{\Phi}^0_a\right]\,,$$

for $k = 1, \ldots, 4$, where $h_4 = G^0$.

The *invariant* quantities $q_{k\ell}$ are given by:

k	q_{k1}	q_{k2}
1	$c_{12}c_{13}$	$-s_{12} - ic_{12}s_{13}$
2	$s_{12}c_{13}$	$c_{12} - is_{12}s_{13}$
3	s_{13}	ic_{13}
4	i	0

The $q_{k\ell}$ are functions of the angles θ_{12} and θ_{13}, where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$.

Since $\widehat{w}_a e^{-i\theta_{23}}$ is a *proper* U(2)-vector, we see that the mass-eigenstate fields are indeed U(2)-invariant fields. Inverting the previous result yields:

$$\Phi_a = \begin{pmatrix} G^+\widehat{v}_a + H^+\widehat{w}_a \\ \dfrac{v}{\sqrt{2}}\widehat{v}_a + \dfrac{1}{\sqrt{2}}\displaystyle\sum_{k=1}^{4}\left(q_{k1}\widehat{v}_a + q_{k2}e^{-i\theta_{23}}\widehat{w}_a\right) h_k \end{pmatrix}.$$

The gauge boson–Higgs boson interactions

$$
\begin{aligned}
\mathscr{L}_{VVH} = {} & \left(g m_W W_\mu^+ W^{\mu\,-} + \frac{g}{2c_W} m_Z Z_\mu Z^\mu\right) \mathrm{Re}(q_{k1}) h_k + e m_W A^\mu (W_\mu^+ G^- + W_\mu^- G^+) \\
& - g m_Z s_W^2 Z^\mu (W_\mu^+ G^- + W_\mu^- G^+) \,,
\end{aligned}
$$

$$
\begin{aligned}
\mathscr{L}_{VVHH} = {} & \left[\tfrac{1}{4} g^2 W_\mu^+ W^{\mu\,-} + \frac{g^2}{8c_W^2} Z_\mu Z^\mu\right] \mathrm{Re}(q_{j1}^* q_{k1} + q_{j2}^* q_{k2})\, h_j h_k \\
& + \left[\tfrac{1}{2} g^2 W_\mu^+ W^{\mu\,-} + e^2 A_\mu A^\mu + \frac{g^2}{c_W^2}\left(\tfrac{1}{2} - s_W^2\right)^2 Z_\mu Z^\mu + \frac{2ge}{c_W}\left(\tfrac{1}{2} - s_W^2\right) A_\mu Z^\mu\right] (G^+ G^- + H^+ H^-) \\
& + \left\{ \left(\tfrac{1}{2} e g A^\mu W_\mu^+ - \frac{g^2 s_W^2}{2c_W} Z^\mu W_\mu^+\right) (q_{k1} G^- + q_{k2}\, e^{-i\theta_{23}} H^-) h_k + \mathrm{h.c.} \right\} ,
\end{aligned}
$$

$$
\begin{aligned}
\mathscr{L}_{VHH} = {} & \frac{g}{4c_W} \mathrm{Im}(q_{j1} q_{k1}^* + q_{j2} q_{k2}^*) Z^\mu h_j \overleftrightarrow{\partial}_\mu\, h_k - \tfrac{1}{2} g \left\{ i W_\mu^+ \left[q_{k1} G^- \overleftrightarrow{\partial}^\mu\, h_k + q_{k2} e^{-i\theta_{23}} H^- \overleftrightarrow{\partial}^\mu\, h_k \right] + \mathrm{h.c.} \right\} \\
& + \left[i e A^\mu + \frac{ig}{c_W}\left(\tfrac{1}{2} - s_W^2\right) Z^\mu \right] (G^+ \overleftrightarrow{\partial}_\mu\, G^- + H^+ \overleftrightarrow{\partial}_\mu\, H^-) \,.
\end{aligned}
$$

The cubic and quartic Higgs couplings

$$\begin{aligned}
\mathscr{L}_{3h} = &-\tfrac{1}{2}v\, h_j h_k h_\ell \Big[q_{j1}q^*_{k1}\mathrm{Re}(q_{\ell 1})Z_1 + q_{j2}q^*_{k2}\,\mathrm{Re}(q_{\ell 1})(Z_3+Z_4) + \mathrm{Re}(q^*_{j1}q_{k2}q_{\ell 2}Z_5\, e^{-2i\theta_{23}}) \\
&\qquad + \mathrm{Re}\Big([2q_{j1}+q^*_{j1}]q^*_{k1}q_{\ell 2}Z_6\, e^{-i\theta_{23}}\Big) + \mathrm{Re}(q^*_{j2}q_{k2}q_{\ell 2}Z_7\, e^{-i\theta_{23}})\Big] \\
&-v\, h_k G^+G^-\Big[\mathrm{Re}(q_{k1})Z_1 + \mathrm{Re}(q_{k2}\, e^{-i\theta_{23}}Z_6)\Big] + v\, h_k H^+H^-\Big[\mathrm{Re}(q_{k1})Z_3 + \mathrm{Re}(q_{k2}\, e^{-i\theta_{23}}Z_7)\Big] \\
&-\tfrac{1}{2}v\, h_k\Big\{G^-H^+\, e^{i\theta_{23}}\Big[q^*_{k2}Z_4 + q_{k2}\, e^{-2i\theta_{23}}Z_5 + 2\mathrm{Re}(q_{k1})Z_6\, e^{-i\theta_{23}}\Big] + \mathrm{h.c.}\Big\}\,,
\end{aligned}$$

$$\begin{aligned}
\mathscr{L}_{4h} = &-\tfrac{1}{8}h_j h_k h_l h_m\Big[q_{j1}q_{k1}q^*_{\ell 1}q^*_{m1}Z_1 + q_{j2}q_{k2}q^*_{\ell 2}q^*_{m2}Z_2 + 2q_{j1}q^*_{k1}q_{\ell 2}q^*_{m2}(Z_3+Z_4) \\
&+2\mathrm{Re}(q^*_{j1}q^*_{k1}q_{\ell 2}q_{m2}Z_5\, e^{-2i\theta_{23}}) + 4\mathrm{Re}(q_{j1}q^*_{k1}q^*_{\ell 1}q_{m2}Z_6\, e^{-i\theta_{23}}) + 4\mathrm{Re}(q^*_{j1}q_{k2}q_{\ell 2}q^*_{m2}Z_7\, e^{-i\theta_{23}})\Big] \\
&-\tfrac{1}{2}h_j h_k G^+G^-\Big[q_{j1}q^*_{k1}Z_1 + q_{j2}q^*_{k2}Z_3 + 2\mathrm{Re}(q_{j1}q_{k2}Z_6\, e^{-i\theta_{23}})\Big] \\
&-\tfrac{1}{2}h_j h_k H^+H^-\Big[q_{j2}q^*_{k2}Z_2 + q_{j1}q^*_{k1}Z_3 + 2\mathrm{Re}(q_{j1}q_{k2}Z_7\, e^{-i\theta_{23}})\Big] \\
&-\tfrac{1}{2}h_j h_k\Big\{G^-H^+\, e^{i\theta_{23}}\Big[q_{j1}q^*_{k2}Z_4 + q^*_{j1}q_{k2}Z_5\, e^{-2i\theta_{23}} + q_{j1}q^*_{k1}Z_6\, e^{-i\theta_{23}} + q_{j2}q^*_{k2}Z_7\, e^{-i\theta_{23}}\Big] + \mathrm{h.c.}\Big\} \\
&-\tfrac{1}{2}Z_1G^+G^-G^+G^- - \tfrac{1}{2}Z_2H^+H^-H^+H^- - (Z_3+Z_4)G^+G^-H^+H^- \\
&-\tfrac{1}{2}(Z_5H^+H^+G^-G^- + Z_5^*H^-H^-G^+G^+) - G^+G^-(Z_6H^+G^- + Z_6^*H^-G^+) - H^+H^-(Z_7H^+G^- + Z_7^*H^-G^+)\,.
\end{aligned}$$

The Higgs-fermion Yukawa couplings

The Yukawa Lagrangian, in terms of the quark mass-eigenstate fields, is:

$$-\mathscr{L}_{\rm Y} = \overline{U}_L \widetilde{\Phi}^0_{\bar a} \eta^U_a U_R + \overline{D}_L K^\dagger \widetilde{\Phi}^-_{\bar a} \eta^U_a U_R + \overline{U}_L K \Phi^+_a \eta^{D\,\dagger}_{\bar a} D_R + \overline{D}_L \Phi^0_a \eta^{D\,\dagger}_{\bar a} D_R + {\rm h.c.}\,,$$

where $\widetilde{\Phi}_{\bar a} \equiv (\widetilde{\Phi}^0\,,\, \widetilde{\Phi}^-) = i\sigma_2 \Phi^*_{\bar a}$ and K is the CKM mixing matrix. The $\eta^{U,D}$ are 3×3 Yukawa coupling matrices. It is convenient to write:

$$\eta^Q_a = \kappa^Q \widehat{v}_a + \rho^Q \widehat{w}_a \quad \Longrightarrow \quad \kappa^Q \equiv \widehat{v}^*_{\bar a} \eta^Q_a \quad {\rm and} \quad \rho^Q \equiv \widehat{w}^*_{\bar a} \eta^Q_a\,, \qquad (Q = U \ {\rm or}\ D)\,.$$

Under a U(2) transformation, κ^Q is invariant, whereas $\rho^Q \to (\det U)\rho^Q$.

By construction, κ^U and κ^D are proportional to the (real non-negative) diagonal quark mass matrices M_U and M_D, respectively, whereas the matrices ρ^U and ρ^D are independent complex 3×3 matrices. In particular,

$$M_U = \frac{v}{\sqrt{2}} \kappa^U = {\rm diag}(m_u\,,\, m_c\,,\, m_t)\,, \qquad M_D = \frac{v}{\sqrt{2}} \kappa^{D\,\dagger} = {\rm diag}(m_d\,,\, m_s\,,\, m_b)\,.$$

The fermion–Higgs boson interactions

The final form for the Yukawa couplings of the mass-eigenstate Higgs bosons and the Goldstone bosons to the quarks is [with $P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$]:

$$-\mathscr{L}_Y = \frac{1}{v}\overline{D}\left\{M_D(q_{k1}P_R + q_{k1}^* P_L) + \frac{v}{\sqrt{2}}\left[q_{k2}\,[e^{i\theta_{23}}\rho^D]^\dagger P_R + q_{k2}^*\, e^{i\theta_{23}}\rho^D P_L\right]\right\}Dh_k$$

$$+\frac{1}{v}\overline{U}\left\{M_U(q_{k1}P_L + q_{k1}^* P_R) + \frac{v}{\sqrt{2}}\left[q_{k2}^*\, e^{i\theta_{23}}\rho^U P_R + q_{k2}\,[e^{i\theta_{23}}\rho^U]^\dagger P_L\right]\right\}Uh_k$$

$$+\left\{\overline{U}\left[K[\rho^D]^\dagger P_R - [\rho^U]^\dagger K P_L\right]DH^+ + \frac{\sqrt{2}}{v}\overline{U}\left[KM_D P_R - M_U K P_L\right]DG^+ + \text{h.c.}\right\}$$

By writing $[\rho^Q]^\dagger H^+ = [\rho^Q e^{i\theta_{23}}]^\dagger [e^{i\theta_{23}}H^+]$, we see that the Higgs-fermion Yukawa couplings depend only on invariant quantities: the diagonal quark mass matrices, $\rho^Q e^{i\theta_{23}}$, and the invariant angles θ_{12} and θ_{13}.

The couplings of the neutral Higgs bosons to quark pairs are generically CP-violating as a result of the complexity of the q_{k2} and the fact that the matrices $e^{i\theta_{23}}\rho^Q$ are not generally either pure real or pure imaginary.

Conditions for neutral Higgs CP-conservation

• $\mathrm{Im}\,(Z_5^* Z_6^2) = \mathrm{Im}\,(Z_5^* Z_7^2) = \mathrm{Im}\,(Z_6^* Z_7) = 0$ [equivalent to conditions first obtained by Lavoura and Silva and by Botella and Silva].

In this case a *real basis* exists in which all potentially complex coefficients of the scalar potential in the Higgs basis are real (as the scalar potential minimum condition fixes $Y_3 = -\frac{1}{2} Z_6 v^2$).

• $Z_5(\rho^Q)^2$, $Z_6\rho^Q$ and $Z_7\rho^Q$ are real matrices ($Q = U$, D and E).

This guarantees that the couplings of the neutral Higgs boson to fermion pairs are CP-invariant.

If the two conditions above are satisfied, then the neutral Higgs bosons are eigenstates of CP, and the only source of CP-violation is the unremovable phase in the CKM matrix that enters via the charged current interactions mediated by either $W^\pm$, $H^\pm$ or $G^\pm$ exchange.

Identifying the CP-odd scalar

There are three neutral Higgs mass-eigenstates: h_1, h_2 and h_3. In the CP-conserving limit, two are CP-even and one is CP-odd. (The fourth scalar, the Goldstone boson G^0 is always CP-odd.) A Higgs scalar is CP-even if $\mathrm{Re}(q_{k1}) \neq 0$.

Suppose that $Z_6 \neq 0$. Then, there are three cases:

1: $s_{13} = \mathrm{Im}(Z_5 e^{-2i\theta_{23}}) = \mathrm{Im}(Z_6 e^{-i\theta_{23}}) = \mathrm{Im}(Z_7 e^{-i\theta_{23}}) = \mathrm{Im}(e^{i\theta_{23}}\rho^Q) = 0$.

k	q_{k1}	q_{k2}	CP
1	c_{12}	$-s_{12}$	$+1$
2	s_{12}	c_{12}	$+1$
3	0	i	-1
4	i	0	-1

2: $s_{12} = \mathrm{Im}(Z_5 e^{-2i\theta_{23}}) = \mathrm{Re}(Z_6 e^{-i\theta_{23}}) = \mathrm{Re}(Z_7 e^{-i\theta_{23}}) = \mathrm{Re}(e^{i\theta_{23}}\rho^Q) = 0.$

k	q_{k1}	q_{k2}	CP
1	c_{13}	$-is_{13}$	$+1$
2	0	1	-1
3	s_{13}	ic_{13}	$+1$
4	i	0	-1

3: $c_{12} = \mathrm{Im}(Z_5 e^{-2i\theta_{23}}) = \mathrm{Re}(Z_6 e^{-i\theta_{23}}) = \mathrm{Re}(Z_7 e^{-i\theta_{23}}) = \mathrm{Re}(e^{i\theta_{23}}\rho^Q) = 0.$

k	q_{k1}	q_{k2}	CP
1	0	1	-1
2	$-c_{13}$	is_{13}	$+1$
3	s_{13}	ic_{13}	$+1$
4	i	0	-1

Note that G^+G^- and H^+H^- can only couple to the CP-even neutral Higgs boson:

$$\mathcal{L}_{\rm int} \ni v h_k \Big\{ G^+G^- \left[\mathrm{Re}(q_{k1})Z_1 + \mathrm{Re}(q_{k2}e^{-i\theta_{23}}Z_6)\right] + H^+H^- \left[\mathrm{Re}(q_{k1})Z_3 + \mathrm{Re}(q_{k2}e^{-i\theta_{23}}Z_7)\right]\Big\} .$$

We see that Higgs sector CP-conservation implies that $\mathrm{Im}(Z_5 e^{-2i\theta_{23}}) = 0$. Assuming $Z_5 \neq 0$, one can eliminate θ_{23} by defining the basis-invariant quantity,

$$\varepsilon_{56} \equiv \frac{\mathrm{Re}(Z_5^* Z_6^2)}{|Z_5||Z_6|^2},$$

since $\mathrm{Im}(Z_5^* Z_6^2) = 0$. Then,

$$\mathrm{Re}(Z_5 e^{-2i\theta_{23}}) = \pm\varepsilon_{56}|Z_5|,$$

where the plus [minus] sign above is taken when $\mathrm{Im}(Z_6 e^{-i\theta_{23}}) = 0$ [$\mathrm{Re}(Z_6 e^{-i\theta_{23}}) = 0$], corresponding to case 1 [cases 2, 3].

Similarly, we can define ε_{57} by replacing Z_6 with Z_7 in the above formulae. The same argument as above implies that $\varepsilon_{56} = \varepsilon_{57}$.

We then can write for the CP-odd mass:

$$m_A^2 = Y_2 + \tfrac{1}{2}(Z_3 + Z_4 - \varepsilon_{56}|Z_5|)v^2 .$$

Suppose that $Z_5 \neq 0$ and $Z_6 = 0$. For simplicity, assume that the three Higgs masses are non-degenerate. Then each possible case has two subcases:

$$\text{(a) } \mathrm{Im}(Z_7 e^{-i\theta_{23}}) = \mathrm{Im}(e^{i\theta_{23}}\rho^Q) = 0$$

$$\text{(b) } \mathrm{Re}(Z_7 e^{-i\theta_{23}}) = \mathrm{Re}(e^{i\theta_{23}}\rho^Q) = 0$$

1: $s_{13} = c_{12} = \mathrm{Im}(Z_5 e^{-2i\theta_{23}}) = 0$

k	q_{k1}	q_{k2}	CP$_a$	CP$_b$
1	0	1	$+1$	-1
2	-1	0	$+1$	$+1$
3	0	i	-1	$+1$
4	i	0	-1	-1

2: $s_{13} = s_{12} = \mathrm{Im}(Z_5 e^{-2i\theta_{23}}) = 0$

k	q_{k1}	q_{k2}	CP$_a$	CP$_b$
1	1	0	$+1$	$+1$
2	0	1	$+1$	-1
3	0	i	-1	$+1$
4	i	0	-1	-1

3: $c_{13} = \mathrm{Im}(Z_5 e^{-2i\overline{\theta}_{23}}) = 0$

k	q_{k1}	$\overline{q}_{k2}$	CP$_a$	CP$_b$
1	0	i	-1	$+1$
2	0	1	$+1$	-1
3	-1	0	$+1$	$+1$
4	i	0	-1	-1

In this last case, we have defined $\overline{\theta}_{23} \equiv \theta_{23} - \theta_{12}$ and $\overline{q}_{k2} \equiv q_{k2} e^{-i\theta_{12}}$. Likewise, the CP$_a$ and CP$_b$ conditions are defined as above but with θ_{23} replaced by $\overline{\theta}_{23}$.

In all three cases where $Z_6 = 0$,

- The scalar h that is CP-even with respect to both CP$_a$ and CP$_b$ has $m_h^2 = Z_1 v^2$ and couplings to itself, gauge bosons, and fermions that are equivalent to those of the SM Higgs boson.

- If $Z_7 \neq 0$, then the squared-mass of the CP-odd scalar A is given by

$$m_A^2 = Y_2 + \tfrac{1}{2}(Z_3 + Z_4 - \varepsilon_{57}|Z_5|)v^2 \,,$$

  and the squared-mass of the second CP-even scalar H is given by

$$m_H^2 = Y_2 + \tfrac{1}{2}(Z_3 + Z_4 + \varepsilon_{57}|Z_5|)v^2 \,.$$

- If $Z_7 = 0$ but $\rho^Q \neq 0$, then one can define an analogous ε_{5Q} that plays the same role as ε_{57}.

A singular point in the parameter space of CP-conserving 2HDMs:

$$Y_3 = Z_6 = Z_7 = 0 \,.$$

One neutral Higgs boson is CP-even, with couplings identical to those of the SM Higgs boson. The other two neutral Higgs bosons have opposite CP quantum numbers, but the Higgs self-interactions and Higgs boson-vector boson interactions do not determine which of these two neutral Higgs bosons is the CP-odd state.

To identify the CP-odd state, examine the Higgs-fermion Yukawa couplings.*

If $e^{i\theta_{23}}\rho^Q$ is a real matrix (CP$_a$), then h_k is CP-odd if $\mathrm{Im}(q_{k2}) \neq 0$.

If $e^{i\theta_{23}}\rho^Q$ is a pure imaginary matrix (CP$_b$), then h_k is CP-odd if $\mathrm{Re}(q_{k2}) \neq 0$.

If $\rho^Q = 0$, then CP$_a$ and CP$_b$ are two equally valid choices for the definition of CP, and one cannot determine which of the two possible neutral Higgs bosons is CP-odd.

*In the final case 3, replace θ_{23} with $\overline{\theta}_{23}$ and q_{k2} with $\overline{q}_{k2}$.

Custodial symmetry in the 2HDM

In the Standard model, the scalar sector exhibits a global SU(2)$_L\times$SU(2)$_R$ symmetry that is violated only by hypercharge gauge interactions and the Higgs-fermion Yukawa couplings. This global symmetry would be exact in the limit of $g'=0$ and $h_t=h_b$. In the custodial symmetric limit the electroweak ρ-parameter,

$$\rho\equiv\frac{m_W^2}{m_Z^2\cos\theta_W}=1\,,$$

to all orders in perturbation theory. In models with only Higgs doublets, with $g'\neq 0$ and $h_t\neq h_b$, radiative corrections generate corrections to the tree-level relation, $\rho=1$.

Pomarol and Vega studied the implications of custodial symmetry for the 2HDM in 1994. They identified two separate realizations, but failed to realize that their two cases were actually related by a change of Higgs basis! Clearly, basis-independent methods can be valuable here!

Define the 2×2 matrices $\mathbb{M}_1$ and $\mathbb{M}_2$, with columns made up of Higgs-basis fields,

$$\mathbb{M}_1 \equiv \left[i\sigma_2 H_1^* \,, \quad H_1 \right] \,, \qquad\qquad \mathbb{M}_2 \equiv \left[i\sigma_2 (e^{i\chi} H_2)^* \,, \quad e^{i\chi} H_2 \right] \,,$$

where χ reflects the phase freedom in defining the Higgs basis. Under a global SU(2)$_L\times$SU(2)$_R$ transformation, $\mathbb{M}_i \to L\mathbb{M}_i R^\dagger$ $(i = 1, 2)$, where L, $R \in \mathrm{SU}(2)$. The vacuum preserves the diagonal SU(2) custodial symmetry (corresponding to $L = R$), since $\langle \mathbb{M}_1 \rangle = (v/\sqrt{2})\mathbb{1}$ and $\langle \mathbb{M}_2 \rangle = 0$. The SU(2)$_L\times$SU(2)$_R$-invariant scalar potential in the Higgs basis has the form:

$$\begin{aligned} \mathcal{V} = {} & \tfrac{1}{2} Y_1 \mathrm{Tr}\, [\mathbb{M}_1^\dagger \mathbb{M}_1] + \tfrac{1}{2} Y_2 \mathrm{Tr}\, [\mathbb{M}_2^\dagger \mathbb{M}_2] + Y_3 e^{-i\chi}\, \mathrm{Tr}\, [\mathbb{M}_1^\dagger \mathbb{M}_2] + \tfrac{1}{8} Z_1 \left(\mathrm{Tr}\, [\mathbb{M}_1^\dagger \mathbb{M}_1] \right)^2 \\ & + \tfrac{1}{8} Z_2 \left(\mathrm{Tr}\, [\mathbb{M}_2^\dagger \mathbb{M}_2] \right)^2 + \tfrac{1}{4} Z_3 \mathrm{Tr}\, [\mathbb{M}_1^\dagger \mathbb{M}_1] \mathrm{Tr}\, [\mathbb{M}_2^\dagger \mathbb{M}_2] + \tfrac{1}{2} \lambda \left(\mathrm{Tr}\, [\mathbb{M}_1^\dagger \mathbb{M}_2] \right)^2 \\ & + \tfrac{1}{2} \left(Z_6 e^{-i\chi}\, \mathrm{Tr}\, [\mathbb{M}_1^\dagger \mathbb{M}_1] + Z_7 e^{-i\chi}\, \mathrm{Tr}\, [\mathbb{M}_2^\dagger \mathbb{M}_2] \right) \mathrm{Tr}\, [\mathbb{M}_1^\dagger \mathbb{M}_2] \,. \end{aligned}$$

For example, $\mathrm{Tr}\, [\mathbb{M}_1^\dagger \mathbb{M}_2] = e^{i\chi} H_1^\dagger H_2 + \mathrm{h.c.}$, etc. Since $\mathcal{V}$ is hermitian, it follows that:

$$\lambda \equiv Z_4 = Z_5 e^{-2i\chi} \,, \qquad \mathrm{Im}(Y_3 e^{-i\chi}) = \mathrm{Im}(Z_6 e^{-i\chi}) = \mathrm{Im}(Z_7 e^{-i\chi}) = 0 \,.$$

Because Z_4 is real, it follows that $\mathrm{Im}(Z_5^* Y_3^2) = \mathrm{Im}(Z_5^* Z_6^2) = \mathrm{Im}(Z_5^* Z_7^2) = 0$. That is, custodial symmetry implies that the Higgs scalar potential is CP-conserving.

Thus, custodial symmetry requires that:

$$\lambda \equiv Z_4 = Z_5 e^{-2i\chi}\,, \qquad \mathrm{Im}(Y_3 e^{-i\chi}) = \mathrm{Im}(Z_6 e^{-i\chi}) = \mathrm{Im}(Z_7 e^{-i\chi}) = 0\,.$$

Consider a real Higgs basis in which all Higgs basis parameters are simultaneously real. If either Z_6 or Z_7 is non-zero, then $\sin\chi = 0$ and it follows that $Z_4 = Z_5$. If $Y_3 = Z_6 = Z_7 = 0$, then $\sin 2\chi = 0$ and it follows that $Z_4 = \pm Z_5$. The corresponding $basis\text{-}independent$ conditions for custodial symmetry are:

$$Z_4 = \begin{cases} \dfrac{\mathrm{Re}(Z_5^* Z_6^2)}{|Z_6|^2} = \epsilon_{56}|Z_5|\,, & \text{if } Z_6 \neq 0\,, \\[2ex] \dfrac{\mathrm{Re}(Z_5^* Z_7^2)}{|Z_7|^2} = \epsilon_{57}|Z_5|\,, & \text{if } Z_7 \neq 0\,, \\[2ex] \pm|Z_5|\,, & \text{if } Y_3 = Z_6 = Z_7 = 0\,. \end{cases}$$

In a real Higgs basis where Z_6 or Z_7 is non-zero, $\epsilon_{56} = \epsilon_{57} = \mathrm{sgn}\ Z_5$, in which case custodial symmetry implies that $Z_4 = Z_5$.† In contrast, if $Y_3 = Z_6 = Z_7 = 0$, then one can transform $H_2 \to iH_2$ and change the sign of Z_5 while maintaining a real Higgs basis. Thus in this latter case, custodial symmetry requires $Z_4 = \pm|Z_5|$.

†The sign of Z_5 is invariant under an O(2) transformation between any two real bases.

The charged Higgs boson mass is given by

$$M_{H^\pm}^2 = Y_2 + \tfrac{1}{2} Z_3 \,.$$

If A^0 is the CP-odd Higgs boson, we previously noted that:

$$m_A^2 = \begin{cases} Y_2 + \frac{1}{2}(Z_3 + Z_4 - \varepsilon_{56}|Z_5|)\,, & \text{if } Z_6 \neq 0\,, \\ Y_2 + \frac{1}{2}(Z_3 + Z_4 - \varepsilon_{57}|Z_5|)\,, & \text{if } Z_7 \neq 0\,. \end{cases}$$

Hence custodial symmetry implies that

$$\boxed{m_{H^\pm} = m_A\,, \qquad \text{if } Z_6 \neq 0 \text{ or } Z_7 \neq 0\,.}$$

If $Y_3 = Z_6 = Z_7 = 0$, then there are two scalars, h_a and h_b with opposite-sign CP and squared masses

$$m_{h_{a,b}}^2 = Y_2 + \tfrac{1}{2} v^2 (Z_3 + Z_4 \mp |Z_5|)\,,$$

in which case custodial symmetry implies that $H^\pm$ is mass-degenerate with either h_a or h_b. However, if $\rho^Q = 0$ as well, then the absolute CP quantum numbers of h_a and h_b are indeterminate.

Of course, in general $\rho^Q \neq 0$. For example, in a Type-II 2HDM,

$$|\rho^D| = \frac{\sqrt{2}M_D \tan\beta}{v}\,, \qquad\qquad |\rho^U| = \frac{\sqrt{2}M_U \cot\beta}{v}\,,$$

where $\tan\beta = v_2/v_1$ in the basis in which the "wrong-Higgs couplings" vanish.

Thus, we examine the consequences of imposing the custodial symmetry on the Higgs-fermion Lagrangian. The end result is:

$$M_U = M_D\,, \qquad\qquad (e^{i\theta_{23}}\rho^D)^\dagger = e^{i\theta_{23}}\rho^U\,,$$

which does *not* impose CP-conservation on the neutral Higgs-fermion interactions (since the latter requires that $e^{i\theta_{23}}\rho^U$, $e^{i\theta_{23}}\rho^D$ are both either real or pure imaginary matrices).

CP violation $\Longrightarrow$ h_a and h_b do not possess definite CP quantum numbers.

CP conservation $\Longrightarrow$ either h_a or h_b can be CP-even, depending on whether $e^{i\theta_{23}}\rho^Q$ is real or pure imaginary.

Thus, for the case of $Y_3 = Z_6 = Z_7 = 0$, imposing the custodial symmetry can yield $m^2_{H^\pm} = m^2_H$, where H is a CP-even Higgs boson! This is the *twisted* scenario of Gerard and Herquet.

Basis-independent computation of S, T and U in the 2HDM

If the custodial symmetry is violated, then one-loop radiative corrections can shift the tree-level result of $\rho = 1$. Denoting $\alpha T \equiv \delta\rho = \rho - 1$, we find that the contribution of a general (possibly CP-violating) Higgs sector to the T parameter is given by the basis independent result:

$$\alpha T = \frac{g^2}{64\pi^2 m_W^2}\left[\sum_{k=1}^{3} |q_{k2}|^2 F(m_k^2, m_{H^\pm}^2) - q_{k1}^2 F(m_i^2, m_j^2)\right] + \mathcal{O}(g'^2)\,, \quad i \neq j \neq k\,,$$

where $m_k \equiv m_{h_k}$ and

$$F(x,y) \equiv \tfrac{1}{2}(x+y) - \frac{xy}{x-y}\ln(x/y)\,, \qquad F(x,x) = 0\,.$$

This result is consistent with a recent computation of Grimus, Lavoura, Ogreid and Osland.

In the custodial symmetric limit (with $g' = 0$), the Higgs contribution to T must vanish:

$$\sum_{k=1}^{3} |q_{k2}|^2 F(m_k^2, m_{H^\pm}^2) - q_{k1}^2 F(m_i^2, m_j^2) = 0\,.$$

One can check that in all of the CP-conserving cases introduced earlier, the above relation is automatically satisfied when $m_{H^\pm} = m_A$. If $Z_6 = Z_7 = 0$ then $m_{H^\pm} = m_H$ is also an allowed solution. As an example, for the case of:

k	q_{k1}	q_{k2}	CP$_a$	CP$_b$
1	0	1	$+1$	-1
2	-1	0	$+1$	$+1$
3	0	i	-1	$+1$

T is proportional to:

$$F(m_1^2, m_{H^\pm}^2) + F(m_3^2, m_{H^\pm}^2) - F(m_1^2, m_3^2) = 0\,,$$

if $H^\pm$ is degenerate either with h_1 or h_3.

Basis-independent formulae for S and U have also been obtained.

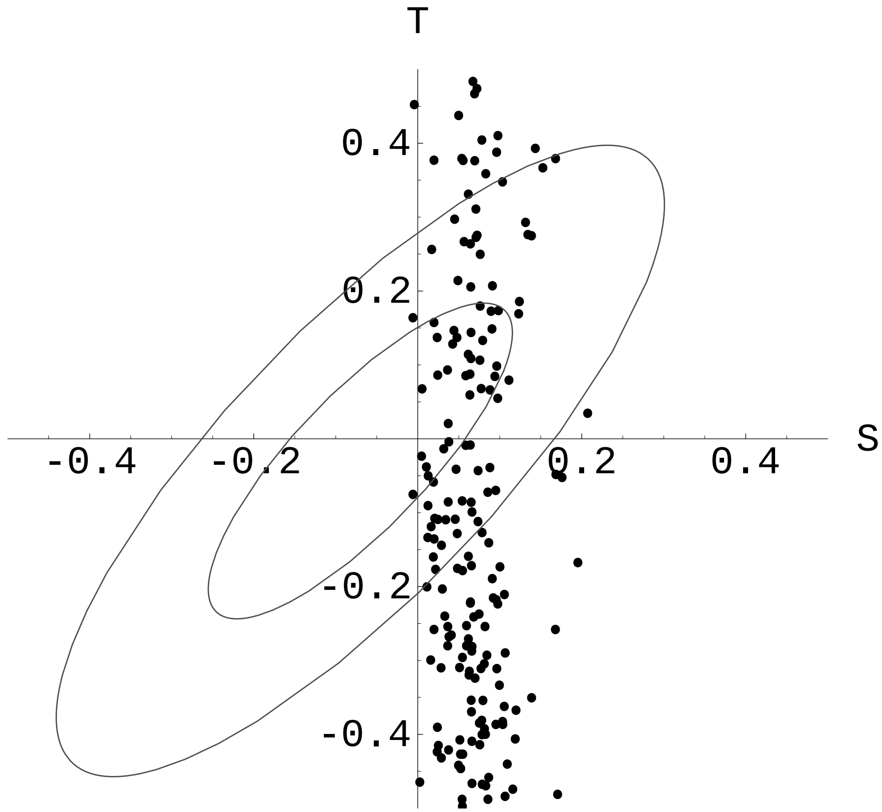
$$S = \frac{1}{\pi m_Z^2}\left[\sum_{k=1}^{3} q_{k1}^2 \mathcal{B}_{22}(m_Z^2; m_Z^2, m_k^2) - m_Z^2 q_{k1}^2 \mathcal{B}_0(m_Z^2; m_Z^2, m_k^2)\right.$$
$$+m_Z^2 \mathcal{B}_0(m_Z^2; m_Z^2, m_\phi^2) + q_{11}^2 \mathcal{B}_{22}(m_Z^2; m_2^2, m_3^2) + q_{21}^2 \mathcal{B}_{22}(m_Z^2; m_1^2, m_3^2)$$
$$\left.+q_{31}^2 \mathcal{B}_{22}(m_Z^2; m_1^2, m_2^2) - \mathcal{B}_{22}(m_Z^2; m_Z^2, m_\phi^2) - \mathcal{B}_{22}(m_Z^2; m_{H\pm}^2, m_{H\pm}^2)\right]$$

Here m_ϕ us a "reference" Higgs mass (often one chooses this mass to be m_Z or the mass of the lightest neutral Higgs boson). Above, we have defined:

$$\mathcal{B}_{22}(q^2; m_1^2, m_2^2) \equiv B_{22}(q^2; m_1^2, m_2^2) - B_{22}(0; m_1^2, m_2^2)\,,$$
$$\mathcal{B}_0(q^2; m_1^2, m_2^2) \equiv B_0(q^2; m_1^2, m_2^2) - B_0(0; m_1^2, m_2^2)\,.$$

where B_{22} and B_0 are the usual Passarino-Veltman one-loop functions. Similarly,

$$S + U = \frac{1}{\pi m_W^2}\left[m_W^2 \mathcal{B}_0(m_W^2; m_W^2, m_\phi^2) - \mathcal{B}_{22}(m_W^2; m_W^2, m_\phi^2) - 2\mathcal{B}_{22}(m_W^2; m_{H\pm}^2, m_{H\pm}^2)\right.$$
$$\left.+\sum_{k=1}^{3} q_{k1}^2 \mathcal{B}_{22}(m_W^2; m_W^2, m_k^2) + |q_{k2}|^2 \mathcal{B}_{22}(m_W^2; m_{H\pm}^2, m_k^2) - q_{k1}^2 m_W^2 \mathcal{B}_0(m_W^2; m_W^2, m_k^2)\right].$$



- The general 2HDM parameters are constrained mainly by T.
- In the decoupling limit, the lightest Higgs mass is constrained in the same manner as in the SM.
- Away from the decoupling limit, regions exist in which the lightest Higgs boson can be significantly heavier than the SM Higgs boson.
- Away from the decoupling limit, the largest allowed mass-splitting between $H^{\pm}$ and the CP-odd Higgs boson occurs before reaching the unitarity limits of the Higgs couplings.

Lessons for future work

- Basis-independent methods provide a powerful technique for studying the theoretical structure of the two-Higgs doublet model.

- These methods provide insight into the conditions for CP-conservation (and violation).

- The basis-independent analysis also clarifies the conditions for custodial symmetry and its breaking

- It is now possible to perform a completely model-independent scan of the 2HDM parameter space. Constraints on this parameter space due to precision electroweak measurements can be obtained, and provide a possible method for avoiding a Higgs boson mass below 200 GeV.